

DOCUMENT RESUME

ED 211 351

SE 035 974

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TITLE Computer-Based Self-Instructional Modules. Final Technical Report.
INSTITUTION Illinois Inst. of Tech., Chicago.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE 77
GRANT NSF-SED-74-14691
NOTE 50p.; Not available in paper copy due to marginal legibility of original document.

EDRS PRICE MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS Chemistry; College Science; *Computer Assisted Instruction; Higher Education; Introductory Courses; *Learning Modules; Mathematics; Physics; Programed Instruction; *Programed Instructional Materials; Science Education

ABSTRACT

Reported is a project involving seven chemists, six mathematicians, and six physicists in the production of computer-based, self-study modules for use in introductory college courses in chemistry, physics, and mathematics. These modules were designed to be used by students and instructors with little or no computer backgrounds, in institutions with modest computer facilities, and were limited to a single major concept or topic. Material included ranged from that covered in a one-hour lecture to a maximum of a single week's study. A complete list of all modules is provided. (FB)

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ED211351

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To appear in the

Proceedings of the NATO Advanced
Study Institute on Computer-based
Science Instruction, Louvain-la-Neuve,
Belgium, 19-30 July 1976

Noordhoff International Publishing, 1977
A. Jones and H. Weinstock, editors

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COMPUTER-BASED SELF-INSTRUCTIONAL MODULES*

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1. HISTORICAL SUMMARY AND MOTIVATION

It is now over twenty years since the development of electronic, high-speed computers, and over ten years since computers became "standard equipment" on most college and university campuses of the western world. Probably in another ten years or less, inexpensive access to computers (or even some form of inexpensive computer itself) should be a feature of most homes. Despite dramatic improvements in technology with attendant reduction of costs, and with the promise of still further improvements, it seems truly amazing, at first glance, that education, - higher education, in particular - has not been greatly affected by the computer, and that generally usable computer-oriented teaching aids are sorely lacking. However, one doesn't have to look too far to see the reasons for this lack of educational use and development of the computer.

First, it should be pointed out that the primary impetus for the birth of the electronic computer resulted from the computational needs associated with the development of the first hydrogen bomb by the Los Alamos Scientific Laboratory in the early 1950's. Thus, the first use of computers was for what is now referred to as "number crunching", the numerical solution of seemingly complex problems in applied mathematics. It was not long, however, before the world of commerce and industry realized the computer's asset as an information processing machine. Combining its ability to

*This work was supported in part by the National Science Foundation and the Exxon Education Foundation.

store, process and retrieve data rapidly, with its equally rapid ability to perform mathematical operations, the computer soon revolutionized the administration of every large business operation.

By the late 1950's, the computer did begin to appear on some U. S. college campuses, particularly on the larger campuses. But given the foregoing historical perspective, it isn't too surprising that practically all use was limited to scientific research computations (for example, by high energy physicists and theoretical chemists) and to purely administrative functions (for example, the printing of pay checks and the keeping of student records). Given the ready availability of basic and applied research funding in those halcyon days, as well as the ever-present "publish or perish" syndrome, there is little wonder that, as an instructional tool, the computer was virtually untouched. Because initial expenditure and the maintenance costs of a computer were borne exclusively by research income and by savings realized in the administrative operation, there was little reason for university administrators to encourage its use for educational development. Surely, such use would only increase the cost of and lengthen the educational process, while at the same time it might limit the computer's access to the "paying customers", those who "really" needed it to perform their duties. The fact that most early campus computers were run in a batch mode, also tended to minimize their educational potential.

Despite the stated problems, some forward-looking faculty members were able to gain access to local computers. Admittedly, some ended up teaching computer programming courses, and thus, never had a real opportunity to investigate the computer's effectiveness as a new educational medium in their chosen scientific disciplines. A select few did, however, begin to adapt the computer to science and mathematics education. As a measure of how few were, indeed, involved, note that in November 1965 the (U.S.) Commission on College Physics sponsored a three day Conference on the Uses of the Computer in Undergraduate Physics Instruction, held at the University of California's Irvine campus, which was attended by only about forty people. Of that number, only fifteen were college physics faculty, one was a chemistry professor, about ten were associated with computer or learning centers, fourteen were employed by computer vendors and three were Commission staff members. The author was told (by one person who attended) that very few of the conference participants had a contribution to make based on actual student participation in computer-related instruction. By contrast, a "Son of Irvine" conference, held at the Illinois Institute of Technology in August 1970 for five days, again with Commission sponsorship, drew almost 400 college faculty, primarily in physics and mathematics.

Clearly then, interest in computer-based science instruction had increased dramatically in only five years, due partially to improved computing capability and access, and partially to the impetus provided by government funding of various computer-based education projects, e.g., a sequence of university-centered regional computer networks [1]. Either with the aid of government support or a dedication to improving the educational process - sometimes both - a still relatively small group of college faculty were introducing the computer into science and mathematics curricula. Yet, the efforts of these "pioneers" were not greatly appreciated by the majority of their colleagues, particularly in research-oriented universities, where the "publish or perish" sword hangs over every untenured faculty member's head. This situation was exacerbated when (inevitably) special funding for computer-related education was terminated. Many colleges found it too expensive to support those programs initiated with external funding, and which, moreover, did not have wide faculty acceptance.

As time progressed, campus access to computers increased. At the same time, faculty interest in educational computing increased too, especially among newer faculty members with practical experience in computer usage. Given an interest in using a computer to enhance the educational process, there still were major obstacles for dedicated faculty to overcome.

Perhaps the first of these obstacles faced is a time limitation for program development. Whether one wishes to adapt a computer program used previously on another campus or to develop one's own ideas, there is an obvious problem in finding the time to create a smooth-running, well documented computer program. Unless a faculty member starts checking-out a program well in advance of its use with students, he soon finds that he is past that point in the curriculum where a particular computer program might be useful. In general, only a very small percentage of instructors find they have the time, as well as the expertise and fortitude, to put an "imported" computer program into a form suitable for student use. It would take several years, in fact, to build a sizable computer library of programs applicable to a particular course offering.

Another time-related obstacle is the problem of integrating computer usage into existing course structure. If the computer is to receive widespread acceptance as an educational tool, it must satisfy two minimal requirements: (1) it must be at least as effective as the medium it replaces in each application; and (2) it must show promise that ultimately it will be no more expensive to use than accepted educational media [2]. The latter requirement precludes using the computer as a supplement to established media, or as a means solely to introduce new

curricular material. Occasionally, either of the above mentioned uses may be feasible, but as a rule they should be avoided. Thus, liberal use of the computer requires restructuring of any given course. Of course, the computational flexibility so gained may lead quite naturally to the introduction of new topics into the curriculum, along with the elimination of others which as a result have become outmoded.

The restructuring of any curriculum is a time-consuming task. The time factor becomes an even greater problem when one discovers a dearth of published textual materials which are computer-related and computer-compatible (due to the accurate perception of publishers that the market for these is too limited). This implies that an instructor must either generate his own printed material or devote extra time in lecture periods to an explanation of the relevant material.

There is yet a third possibility an instructor might consider in attempting to overcome the lack of suitable written materials for computer usage. He could utilize the computer itself to present the relevant pedagogy and to provide instructions for use of computational programs. This "solution", however, serves mainly to point out another major obstacle to greater acceptance of computer-based education: lack of a sufficient supply of terminals, in particular, interactive ones. If those faculty who are attempting to "infect" their colleagues with a sense of the computer's educational utility are only modestly successful, one might often find such "conversions" meaningless when the demand for terminal-time far outstrips its availability. In fact, it seems safe to say that most faculty already using the computer with their students have been limited to doing so with classes of relatively small size. The problem of terminal access for large classes becomes even more acute when lengthy interactive dialogues are involved. Despite recent and expected reductions in the cost of interactive terminals, the limitations of terminal access promise to be a major obstacle for some time to come.

In summary then, educational computer use has been limited primarily by a lack of (1) readily available transportable programs, (2) suitable curricular materials, and (3) sufficient terminal access, particularly where interactive access is desired. This author has attempted to circumvent these obstacles by organizing a collaborative effort among a selected group of six individuals in each of the disciplines: chemistry, physics and mathematics. This project has been developed with support from the National Science Foundation, under its College Faculty Workshop program, and has received additional support from the Exxon Education Foundation for disseminating the resulting computer-based modules.

2. MODULE DEVELOPMENT

The basic goal of this joint effort was to produce a sequence of individual modules, each covering a specific topic at the introductory college level in chemistry, physics or mathematics. A module presents a body of information normally covered in one chapter of a textbook. Unlike most textbooks, however, each module may be used independently, i.e., it doesn't depend specifically on the material presented in any other module in a given series. There are, however, prerequisites stated for most modules, but these may be satisfied in a variety of ways using other educational media.

The motivation in producing these modules is to permit selected use of computer-based educational materials without forcing instructors to adopt radically new curricula with heavy emphasis on computer-based learning. Perhaps most importantly, neither the instructor nor his students need be knowledgeable about computer programming in using a module; and all computer-related exercises can be performed on quite modest minicomputer systems with only alphanumeric printing terminals as output devices. This minimum hardware requirement doesn't preclude an embellishment of programs on more sophisticated operating systems with continuous graphic capability. Furthermore, a section of optional problems in each module contains some designed to challenge those students with programming ability.

3. NATURE OF A MODULE

Before presenting an outline of a module in each of the three project disciplines, it is necessary to review the general structure and educational philosophy imposed. To a large extent, the author was influenced by an "early" monograph entitled "Introductory Computer-based Mechanics" by A. M. Bork, A. Luehrmann and J. H. Robson [3], which was comprised of a Student Manual and a Teacher's Guide. This nomenclature has been retained in the current project, with the modification that the designation Student Manual does not appear in print. Thus, a student refers only to the "module on ----". For purposes of differentiation in the current discourse, the titles, Student Manual and Teacher's Guide, will be used.

Is a Student Manual nothing more than a rewrite of a textbook chapter with occasional directives to use a computer program? The answer to this rhetorical question is a resounding no! This must be so, of necessity, if one is to prevent use of the computer as an addition to existing curricula (even if that additional use has been suitably integrated). The addition of computer exercises

must result in a corresponding restructuring of the written presentation of relevant material.

The Student Manuals have been written somewhat in the form of a dialogue, with liberal use of the second person, e.g., "you should find an optimum path such that ----". The main body of a Student Manual, the DISCUSSION section, requires a student to carry out various exercises before continuing on with the textual narrative. An exercise may involve some simple (hand) calculations, the acquisition of auxiliary information, interpretation of some previous discussion, or access to a computer program, with subsequent interpretation of the resulting output. Certainly, the computer is utilized in many of the exercises, but never (intentionally) when a point may be illustrated more effectively by reference to another medium. It should be clear from the above, that a module demands active student involvement by a variety of means. What (hopefully) will become more evident from later examples, is that this active involvement is discovery oriented. By performing the various hand calculations and computer exercises, the student is expected to "discover" the relevant mathematical and physical laws which might otherwise be stated as fact in a standard text, i.e., the student becomes an active (as opposed to a passive) participant in the learning process.

In keeping sight of the goal to minimize terminal access and CPU requirements, while maximizing the information conveyed, instructions on using a particular program and questions involving interpretation of output, appear only in the text of the module. The computer is utilized primarily for (1) simulations, (2) complex or otherwise tedious calculations, and (3) the generation of graphic or tabular output. Operations involved in the above uses are basically computational. Output is quite often displayed in graphic form, with the understanding that students will come away from a terminal with hard copy in hand. Thus, a student may, at his discretion, review both the output obtained and the module text, integrating them as he proceeds. Typically, a module should require no more than 1 to 1½ hours of total terminal access (at a transmission rate of about 300 baud). However, to be most effective, this total time should generally be the sum of a few individual sessions at a terminal.

4. INTEGRATING MODULES INTO THE CURRICULUM

Assuming active student involvement with each module as described, does a particular module serve only to replace a textbook chapter covering the same general subject matter? The answer to this question may vary, dependent somewhat on the module in question, and somewhat on the attitude of the instructor

using it. At the very least, a computer-based module can certainly be used as a replacement for (or less desirably as a supplement to) a textbook chapter. The only real justification for making such a substitution is that the computer-related exercises and associated discussion provide the student with a deeper insight into the relevant subject matter.

As a first approximation, the module concept is designed to circumvent the obstacles to computer-based education mentioned earlier. It does this by offering an easy-to-use, inexpensive alternative to a (mostly mythical) computer-oriented textbook which requires more computer access than most instructors care to assign, and perhaps more access than is, in fact, feasible. Moreover, the modules are written in a form suitable for application to a personalized (self-paced) system of instruction. While it might be necessary to add conventional, non-computer-oriented, self-paced textual materials to existing modules for completeness, the computer-based modules of the current project require no modification for application to a self-paced curriculum. For example, the author has found it quite convenient on two separate occasions when other duties required absences of about one week from classes, to assign a module based on material not covered in lecture. Subsequent student testing on both occasions yielded average grades higher than those obtained for subject matter presented via lecture and a conventional textbook. While the number of students involved in these two "trials" is not high enough to warrant the heralding of a major breakthrough in educational design, the author was markedly impressed by the fact that the standard deviations from the almost perfect average grades, were exceedingly low. Put more simply, almost everyone had perfect or near-perfect grades for the short quizzes based on the two modules used.

Still another important application of some modules, is to integrate them with conventional laboratory experiments in the physical sciences. While it would be folly to use computer simulations to totally replace existing experiments, the simulations can help to make more efficient use of laboratory time and equipment, and once again, to offer students greater insight into the physical laws governing their observations. The Geometric Optics physics module has been student-tested successfully with this particular approach. While not all modules lend themselves so well to integration with true laboratory experience, this type of application helps to illustrate the ultimate need to restructure educational methods and curricula in order to reap the greatest rewards; educationally and economically, from recent advances in all areas of educational technology.

The various approaches to module use serve to show the flexibility gained by offering a collection of independent,

self-contained materials which are reliant on no particular textbook or curriculum. Because an attempt was made to achieve a high degree of universality, many existing modules contain optional sections to accommodate those students in curricula demanding a higher level of mathematical sophistication. To be sure, no module can be truly universal in its application, and each reflects to a degree the philosophy and student clientele of the individual author(s), as modified by the editor for a given academic discipline. Nevertheless, many of the modules in their current revised form have benefitted from student-testing at more than one institution of higher learning and from subsequent group discussions, both intra- and interdisciplinary ones, among the module project authors.

5. STRUCTURE OF A MODULE

5.1 Student Manual

The Student Manual of all modules contains each of the following headings:

OBJECTIVES
PREREQUISITES
INTRODUCTION
DISCUSSION
REFERENCES
PROBLEMS
APPENDIX

The OBJECTIVES section is generally less than a page in length. It must unambiguously state just what knowledge and techniques the student should have mastered upon completion of the module. While sometimes there may be a few descriptive sentences stating general objectives, there is always a listing of specific objectives in behavioral form, e.g., "Upon completion of this module, you should be able to do the following: 1.---". An example of module objectives, taken from page 1 of Geometric Optics, is presented in Fig. 1. It should be noted, however, that the particular two column, seemingly wasteful [4] printing format is not peculiar to all modules.

Although Fig. 1 indicates that there are no PREREQUISITES needed for use of the Geometric Optics module, this is not often the case. In general, the PREREQUISITES indicate the mathematical background and previous knowledge required to insure successful comprehension of the subject matter to be presented. On occasion, a short pretest is offered, with answers provided. If a student has difficulty in arriving at the correct answers, he is encouraged

OBJECTIVES

This module uses four computer programs to examine the properties of light rays as they cross from one medium to another. It will help explain why things seem to bend going into water, and ultimately, how lenses affect light.

Upon completion of this module, you should be able to do the following:

1. State the principle of "least time" for light. (Fermat's principle)
2. Given an interface between two media having different indices of refraction,

predict the behavior of a light ray crossing that interface.

3. State Snell's Law.
4. Given an interface, determine whether a light ray will be refracted or undergo total internal reflection.
5. Correctly use principal ray diagrams to predict the behavior of light through one or two thin lenses of given focal length and position with respect to some object.

PREREQUISITES

None

Figure 1. OBJECTIVES and PREREQUISITES for Geometric Optics module

to study the related prerequisite topics before proceeding with the main body of the module.

The INTRODUCTION is also generally brief, typically one to two pages in length. Its function is fundamentally to review the requisite background information and to provide an interesting lead-in, to "set the stage", for the new material to be covered in the DISCUSSION section which follows.

In essence, the DISCUSSION has already been covered to a major extent in a previous part of this exposition, namely that entitled "NATURE OF A MODULE". The DISCUSSION section is, indeed, the heart of the module, the surrogate textbook chapter regularly requiring active student participation in the discovery of relationships and ideas in the physical sciences and mathematics. In the DISCUSSION, a student is occasionally referred to other educational media for topics in which the computer can not be used as effectively or in which the other media are complementary to the computer.

With only a limited number of exceptions (all in mathematics), students do not do their own programming in performing the computer-related exercises in the DISCUSSION section. While this means that students must utilize "canned" programs, the text explicitly describes exactly what mathematical and other operations the computer is performing. Occasionally, fully documented examples are given, along with sample computer input and output. Although a range of input parameters is often suggested, a student must give serious consideration to using the most judicious parameters in attempting to answer the questions posed. Thus, even with canned routines, the student's role is more than that of a passive observer to the machinations of the computer. Additionally, he is presented with a tool which enables him to see the effects of changing a variety of parameters over wide ranges. He doesn't need to be limited to a few "representative" illustrations in a textbook or, in some cases, by the constraints of time and equipment in a laboratory situation.

The remaining sections of a Student Manual: REFERENCES, PROBLEMS and APPENDIX, do not differ greatly from their textbook counterparts. Corresponding chapters in a large number of popular textbooks at various levels is presented under REFERENCES. Unlike the DISCUSSION section in which a student is required to do all the exercises listed, the PROBLEM section contains a large body of optional problems to be assigned at the discretion of the instructor. However, as with the DISCUSSION, not all the problems involve the computer. A new feature, alluded to earlier, is that some problems require the application of programming skills. The APPENDIX is used primarily to present well-documented flow charts of all the canned programs embodied in the module.

5.2 Teacher's Guide

Put very simply, the Teacher's Guide provides all the documentation needed to permit students to execute all the programs in the Student Manual, and additionally lists the solutions to all exercises and problems. It also discusses the integration of the module into various existing curricula. The major headings are:

EDUCATIONAL OBJECTIVES
IMPLEMENTATION
SOFTWARE
SOLUTIONS

The EDUCATIONAL OBJECTIVES relate the level of achievement expected of students and the modes by which this is to be accomplished. The IMPLEMENTATION section offers suggestions on where the module can be inserted within the framework of various courses, the total time to be set aside for students to complete

the module, and how much time to allot for computer terminal access. In this section too, there may be a discussion of what optional sections of the Student Manual to use with curricula and students at different levels. If deemed appropriate, auxiliary educational materials are suggested.

The SOFTWARE section includes program listings in relatively unsophisticated BASIC, and ideally, in FORTRAN IV as well. The lack of programming sophistication is deliberate, in keeping with the goal of having programs which can run "as is" on the most modest of minicomputers and (BASIC) programmable calculators. The choice of the BASIC and FORTRAN programming languages implies no particular endorsement of them as being optimum, but rather implies the belief that they are the two languages most readily available. Sometimes included under the SOFTWARE section are typical sample output runs, particularly in those cases for which such output has not been provided in the Student Manual.

6. MODULES IN REALITY

The foregoing exposition was deliberately presented in a dispassionate manner. This was done to put forward a set of ideals to be embodied in the modules which were to be produced. At this point, I just as deliberately switch to a more personal approach in order to lay greater emphasis on the subjective nature of the viewpoints to be expressed in evaluating the finished (or almost finished) products and their degree of success in actual use by students.

Before presenting a few specific examples of some of those finished products, i.e., the modules, as well as discussing the common approaches and some of the common problems faced within each of the three academic disciplines, I wish to describe the manner in which this collaborative project was carried out.

Having decided upon the concept of a modular format as a means for encouraging wider use of computer-based education, it was quite obvious to me that even in dealing only with my own field of physics, the task was too monumental for one individual, or even two, to make significant headway. Furthermore, despite a vast collection of computer programs and rudimentary lesson units (in physics, chemistry and mathematics) accruing from a number of College Teacher Summer Institutes and similar activities I had directed or been associated with, I had no faculty colleagues seriously interested in joining me in a full-scale effort to more fully develop and test the material already on hand. The importance of testing can not be stressed too much, because some of the programs and lesson units written by Summer Institute participants

were not even implemented and used by all the faculty authors when they returned to their home institutions. Since both more manpower was required and the material produced had to be transportable to institutions with differing computational facilities, it seemed quite natural to recruit individuals on a national basis who had already exhibited a strong interest in educational computer use and an ability to write with some clarity.

Each summer since 1974 there has been a two or three week combined workshop session for the five participants and one group leader in all three disciplines. There also have been briefer midyear discipline-oriented workshop sessions in 1975 and 1976 held in conjunction with national professional society meetings. These midyear sessions were organized not only for the purpose of reviewing recent progress, but also for the purpose of disseminating information on our efforts and seeking critical comments from interested parties.

At first, we engaged three consultants whose area of expertise was not necessarily associated with computing, but rather with modularized teaching units and self-paced instruction. After hearing from these consultants, we arrived by consensus at the structural form of the Student Manual and Teacher's Guide described earlier. We furthermore resolved to maintain, as much as reasonably possible, an interdisciplinary approach to our efforts. This is most evident in some of our mathematics modules which deal with non-analytic methods of solution (e.g., numerical integration and numeral methods of solving differential equations) appearing in many of the simulation programs of the chemistry and physics modules. Additionally, the mathematics modules have exercises and problems which relate to realistic situations experienced in the physical sciences. To a lesser extent there is some interplay between physics and chemistry in the areas of thermodynamics, statistical properties and electrostatics.

The task that we had set for ourselves being a major one, even in light of our total number - eighteen in all - and our collective commitment to computer-based science instruction, it was resolutely determined to "stand on the shoulders of others" in applying ourselves. In other words, we wished to adopt and adapt as many existing computer programs which we found available and which satisfied our needs. Completely new programs were to be written only when nothing appropriate was found in the public domain or in the private collections of the participants.

In keeping with our concern for flexibility and for offering computer-oriented instruction in "small bites", we wanted each module to encompass no more than one week of total student activity. In some cases, when it was found impossible to meet that criterion, a given module was broken up into two or more

units, each of which is required to consume no more than one week's effort. Occasionally also, a module is formed into individual units merely for the purpose of delineating various subtopics. Generally, a unit is not intended to be independent of the other units in a particular module. However, more advanced topics or optional material is sometimes placed into the last unit of a module where it may or may not be assigned according to the discretion of a particular instructor. For example, in the mathematics module entitled "Elementary Numerical Solutions for Ordinary Differential Equations", there are three units, each, in turn, dealing with a higher level of approximation. It is entirely possible that many instructors will be satisfied with (or be forced by time constraints to be satisfied in) assigning only the first two units, which cover numerical methods up to and including the fourth order.

A module is ordinarily the direct result of the efforts of one or two individuals, the only exceptions occurring in the chemistry area in which as many as four of the group's members contributed one or more units to a specific module. Nevertheless, every module in each discipline received critical input and scrutiny from (at least) the entire membership of that discipline at the various project workshop sessions. As a result, I feel that the current revised versions of most of our modules have been significantly improved over the corresponding preliminary versions. I make this bold statement despite the well-known belief that a camel is a horse designed by a committee.

A full set of module abstracts for all disciplines appears in the Appendix to this article. A few of these modules are not yet available for testing. Below I describe in greater detail a few representative ones and describe some general features peculiar to the efforts of each discipline.

6.1 Chemistry

The chemists have been the most prolific and best organized of the three discipline groups. This is a result partly due to the efforts of a seventh member of this group during the original three week workshop session, and partly due to a greater collective spirit on the part of chemists in general, and these chemists in particular. By far the major reason for this superlative effort is the dedication and hard work of the group's leader, Professor Cynthia Jameson.

At the moment, the totality of the chemistry modules are felt by their authors to comprise just about all the topics of an introductory one year college chemistry course which can be realistically adapted to computer-based education of the type

being used. Since, in their opinion, this comes rather close to the totality of topics normally covered, they now would appear to be in a position to offer a self-paced, computer-oriented course. They have strengthened this possibility by the addition of module-like, modular appendices. These brief units have no computer-related elements, but provide expositions and practice on the understanding of pH and on related mathematical operations which are fundamental to any chemistry curriculum. The three initial modular appendices are entitled "Exponential Numbers and Logarithms", "Use of Conversion Factors" and "pH." Recently a fourth modular appendix on the "Naming of Chemical Compounds" has been added, and this does use a computer game to aid the student.

As an example of a module of major importance, and one which relies quite heavily on the computer in a variety of applications, I wish now to review in some detail the module on "Chemical Equilibrium", which is composed of five individual units.

Unit 1, entitled "Introduction to Chemical Equilibrium" attempts to have students discover the concept of equilibrium from simulated empirical data. As such, it is essential that students encounter the module prior to any discussion of this topic in a classroom lecture (if such lectures are presented at all). The EDUCATIONAL OBJECTIVES in the Teacher's Guide state that upon completion of this unit, a student should have (1) a qualitative understanding of chemical equilibrium, (2) an ability to write an equilibrium expression for any chemical reaction and (3) an ability to predict the direction in which a reaction will proceed in order to achieve equilibrium.

The Student Manual begins with a somewhat more explicit statement of OBJECTIVES than those stated just above. The PREREQUISITES are stated simply as an understanding of (1) the concept and (2) the stoichiometry of chemical reactions. To test this understanding, a PRE-TEST containing two multiple part questions is presented, followed by a listing of the answers. The INTRODUCTION then begins by sending the student to a computer terminal to execute the program EQSIM.

After calling the program, the student is greeted by the output shown in Figs. 2a and 2b, with the student merely entering three values of time in minutes on separate occasions. After each set of student inputs and each set of graphical responses (denoting gas molecules A, B and C moving randomly in a container), he is asked the two questions, both in the computer output and in the module text, "How do these pictures differ? How are they similar?" The text goes on to provide a discussion which points out the correct observations that should have been made by the student. The major objective of EQSIM is to give the student an

♦♦THE CONCEPT OF EQUILIBRIUM♦♦

THE PURPOSE OF THIS PROGRAM IS TO AID YOU IN UNDERSTANDING THE CONCEPT OF EQUILIBRIUM. THE GAS EQUUS REACTION DISCUSSED IS



FOUR PICTURES WILL BE SHOWN WHICH WILL REPRESENT THE SYSTEM AS TIME CHANGES, BEGINNING WITH TIME, $T = 0$. CHOOSE THREE TIMES (IN MINUTES AFTER TIME ZERO) THAT YOU WOULD LIKE TO SEE, AND ENTER THESE TIMES (SEPARATED BY COMMAS) AFTER THE QUESTION MARK APPEARS -- E.G. ? 2,4,6

PLEASE ENTER THREE TIMES IN ASCENDING ORDER?10,15,20

TIME IS 0 MINUTES.

[illegible]

TIME IS 10 MINUTES.

Figure 2a. Partial output for the initial half of the program EQSIM.

NOW LET US CONSIDER STARTING WITH SUBSTANCES
B AND C AND OBSERVE HOW THE SYSTEM ATTAINS EQUILIBRIUM.
AS BEFORE, CHOOSE THREE TIMES YOU WISH TO OBSERVE.

PLEASE ENTER THREE TIMES IN ASCENDING ORDER?10,15,20

```

*****
* C C C C C C C B B B C B *
*                               *
*   B C B   C   B   C   C   *
*   B B   B   C   C   C   *
*   C   C   B   C   C   C   *
*   B B   C   C   C   C   *
*   B   C   C   C   C   B *
*   C B B   B   C   C   *
*   B   B   C   C   C   *
*****

```

TIME IS 0 MINUTES.

```

*****
*   C   B   B   C   C   *
*   C   C   C   C   C   *
*   C   C   C   C   C   *
*   C   B   C   C   C   *
*   C   C   C   C   C   *
*   B   A   B   B   A   *
*   B   C   C   C   C   *
*   B   C   C   C   C   *
*****

```

TIME IS 10 MINUTES.

Figure 2b. Partial output for the final half of the program EQSIM.

intuitive feeling for dynamic equilibrium. By counting the
gaseous molecules of types A, B and C, he is expected to note
the similarities and differences between the different pictures
as the equilibrium state is approached from both sides. (In the
oral presentation of this paper, some observers felt that the

THIS PROGRAM IS DESIGNED TO HELP THE
STUDENT EMPIRICALLY DERIVE THE
EQUILIBRIUM EXPRESSION AND GAIN INSIGHT
INTO THE CONCEPT OF CHEMICAL EQUILIBRIUM.
GIVEN THE FOLLOWING CHEMICAL REACTION:



THE FOLLOWING EXPERIMENTAL RESULTS WERE
OBSERVED. (CONC. IN MOLES/LITER)

A(INITIAL)	A(EQ)	B(EQ)	C(EQ)
10.0	2.679	7.320	7.320
7.0	1.508	5.492	5.492
5.0	.858	4.142	4.142
3.0	.351	2.649	2.649
2.0	.168	1.832	1.832

IT IS POSSIBLE TO FIND A RELATION AMONG
THE EQUILIBRIUM CONCS. WHICH DOES NOT
CHANGE AS A(INITIAL) IS CHANGED. TRY
VARIOUS COMBINATIONS OF THE EQUIL. CONCS.
AS FUNCTIONS OF A(INITIAL) TO FIND AN
EXPRESSION WHICH IS INVARIANT.

FOR EXAMPLE: A(EQ) VERSUS A(INITIAL)

```

I      0
I
I
I      0
I
I      0
I
I      0
I
I
IO
*****
A(INITIAL)

```

OBTAINING THIS CHANGES SHOWING THAT
A(EQ) IS DEPENDENT UPON A(INITIAL)

FROM THE FOLLOWING TABLE SELECT THE
EXPRESSION YOU WOULD EXPECT TO BE
INDEPENDENT OF A(INITIAL).

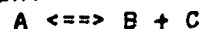
- | | | | |
|-----|---------|-----|---------|
| (1) | (A*B) | (2) | (B**2) |
| (3) | (C*B)/A | (4) | (C*B) |
| (5) | (C/A) | (6) | (C*B+A) |

Figure 3a. First part of output for program KEQ.

After the introductory discourse, the student is requested to run LECHAT, a program which simulates the gas phase reaction $A \rightleftharpoons B + C$. Figs. 4a and 4b show part of a typical program run in which the student can observe the effect of volume change (at constant temperature) on the relative equilibrium concentration of the gaseous molecules. Not shown is similar output which shows the equilibrium concentration for each initial number of molecules (for A, B and C) inputted by the student. Back in the text of this unit, the student is asked to fill in a table which summarizes the system's response, i.e., the changes in the numbers of A, B and C, to the various applied stresses, such as adding more of any constituent molecule or changing the pressure (by increasing or decreasing the volume at constant temperature). The remaining DISCUSSION follows conventional lines with problems interspersed and at the end, It is intended that this unit be used prior to a possible (but really unnecessary) lecture on the same subject.

** LE CHATELIER'S PRINCIPLE **

DO YOU WANT AN INTRODUCTION? (YES OR NO)? YES
THIS PROGRAM ILLUSTRATES LE CHATELIER'S PRINCIPLE
BY ALLOWING YOU TO VARY THE PARAMETERS OF A SYSTEM AT
EQUILIBRIUM, NOTING THE EFFECTS. AN INITIAL SYSTEM IS
SHOWN WITH MOLECULES A, B, AND C IN A ONE-LITER FLASK
FOR THE REACTION:



WHERE A, B, AND C ARE GASEOUS MOLECULES.

```
*****
*      C  A  C      *
*      B      C      *
*      A      B      *
*  A  A      C  C      *
*  B  B      B  A      *
*      A      C      *
*      C  C  A      B  *
*      A      B      *
*****
DO YOU WANT TO SEE THE EFFECTS OF CHANGING THE VOLUME? YES
```

CHOOSE A VOLUME BETWEEN 0.5 AND 1.5 LITERS.
?.5

```
*****
*  A  A      A      *
*      A      A      *
*      A  C  A  C      *
*      A      B      *
*  A      C      C  A      *
*  B      C      B      *
*      B      B  C      *
*  A  C      B      *
*      A      B      *
*      B      B  C  A      *
*****
DO YOU WANT TO SEE ANOTHER VOLUME? YES
CHOOSE A VOLUME BETWEEN 0.5 AND 1.5 LITERS.
?.125
```

```
*****
*      A      C      A      B      *
*  A      C      C      B      *
*      B      A      A      C      *
*      B      B      C      B      *
*      B      B      C      C      *
*      C      B      A      C      *
*  A      B      C      C      *
*      C      A      *
*****
DO YOU WANT TO SEE ANOTHER VOLUME? NO
```

Figure 4b. Student generated output for the program LECHAT.

Because of spatial limitations, it is not possible to review the remaining units of Chemical Equilibrium in depth, and the abstracts presented in the Appendix will have to suffice. A few general comments are, however, in order. In Unit 3, "Chemical Equilibrium Calculations," there are three computer programs utilized. The first employs a trial-and-error approach to finding equilibrium concentrations for problems that the student is taught to solve by analytic means as well. The second program uses the same approach for the student's choice of reaction and

initial conditions on systems too complex to handle algebraically. The third program involves the generation of a quiz, with random number generation utilized to provide personalized tests. A student works on the problems off-line, and then inputs his answers and quiz code number to the computer at a later time for grading.

The last mentioned program is a perfectly valid application of computer-based education, but it does fall outside the objectives of the module project itself. While it is a reasonably efficient use of the computer for drill-and-practice work, it does require a non-trivial amount of terminal printing time, and for many computer environments terminal accessibility is the limiting feature, a situation that these modules are designed to alleviate. A still even more non-essential computer use comes in Unit 5, "Heterogeneous Ionic Equilibria," in which the computer merely prints out a page of tabular and graphical output (upon calling and running the program). There is no student input to the program at all and every student has the same data presented. The Teacher's Guide for this unit states that the output is designed to give a student the feeling of having performed a real experiment. However, I question whether a page of identical text in the Student Manual is any less valid in attempting to convey this feeling.

It might be noted also that introductory text prior to each simulation could more economically (in some instances) be embodied in the Student Manuals, or at least offered only as an option for those students who occasionally might find themselves at a terminal without the benefit of a Student Manual to consult. Many such introductory remarks were put in when programs were first written and prior to the time when modules were in typed form. As work on this project continues, dialogue found superfluous will be expunged from the final versions of our programs.

The critical comments just made above, should not be construed as a negation of the laudatory statements made at the beginning of this section. Rather, these comments have been made to show how the actual modules deviate from the stated goals of the project, and to indicate that we have not yet produced our final products. These remarks apply equally well to the other two project disciplines. In general, I am quite pleased with how far we have come in a period of only two years. This could have occurred only with a strong effort of all participants, particularly when working on their own between group workshop sessions.

6.2 Physics

Probably the major problem faced in producing universally acceptable curricular materials for an introductory physics course, is the great variation that exists in course offerings, even within a given university. A sequence of courses covering the same major topics may vary anywhere from three credit-hours per week for one or two semesters to four credit-hours per week for four semesters; the mathematical backgrounds of the students may vary from an inability to do simple algebra to proficiency in analytic geometry and calculus; and the career interests of the students span an incredible range including poetry, nursing, electronic repair, photography, business, mortuary science, psychology, biological science, physical science, engineering, etc. Even allowing for variation in level of presentation, there is probably no general consensus on which topics to cover after dealing with the basic ones of mechanics, electricity and magnetism, and the conservation laws of energy and momentum. Even within those areas there is a wide latitude of approaches. We decided as a primary criterion to choose subject matter which could be enhanced best by computer simulations, and worried about the universality of the material as a secondary consideration only. By adopting the "student discovery" type of approach, we felt that all students would benefit, even those with the mathematical sophistication to derive most relevant analytic expressions. Optional sections of Student Manuals have been included to satisfy the needs of students requiring greater detail and having a more highly developed mathematical background.

Almost all physics modules employ simulations designed to provide insight, and some involve "games that teach." Two of the modules, "Wave Motion" and "Electrostatics" use the computer primarily as a graphing tool: the former to plot the addition of sine waves; the latter to plot electric field lines and (two-dimensional) equipotential surfaces for arbitrary electric charge distributions. As an example of the computer as a simulation device of unusual power, i.e., as a tool offering exquisite insight, I review now the module entitled "Statistical Properties and the Behavior of Gases."

This module is broken into three short units, each dealing with a different (but related) topic. Total terminal time required, has been found from experience to be about one hour. There are no prerequisites, yet the major EDUCATIONAL OBJECTIVE is to introduce the statistical foundations of thermodynamics and kinetic theory into introductory physics. This is accomplished by emphasizing the dependence of macroscopic properties of systems on the randomness of those systems. Additional factors such as energy conservation are introduced, and their consequences are investigated. Because many macroscopic properties are independent

of the details of the model chosen to describe the microscopic structure of a system, extremely simple models may be used to generate physically realistic simulations.

The INTRODUCTION of Unit 1, "Random Walk and Diffusion" provides an elementary discussion of randomness as applied to coin tossing and dice rolling. The DISCUSSION section then continues this discourse by applying coin tossing to the classic one-dimensional random walk problem. It goes on to relate this to a computer program RAND which simulates exactly this problem. The program's algorithm is fully explained and the technique itself is described as a form of Monte Carlo technique to be utilized in subsequent programs.

In a first exercise with RAND, the student runs the program for different walk lengths, from 10 to 200 steps of one unit each, 200 times for each length, and with a probability of 0.5 for a move to the right. Figure 5 illustrates a typical output histogram showing the frequency of the various endpoints reached. The student may also request output in tabular form.

LENGTH OF EACH WALK? (BETWEEN 2 AND 100). ?10
PROBABILITY OF A MOVE TO THE RIGHT? (BETWEEN 0 AND 1)?0.5
NUMBER OF RANDOM WALKS TO RUN? (FROM 1 TO 9999)?200

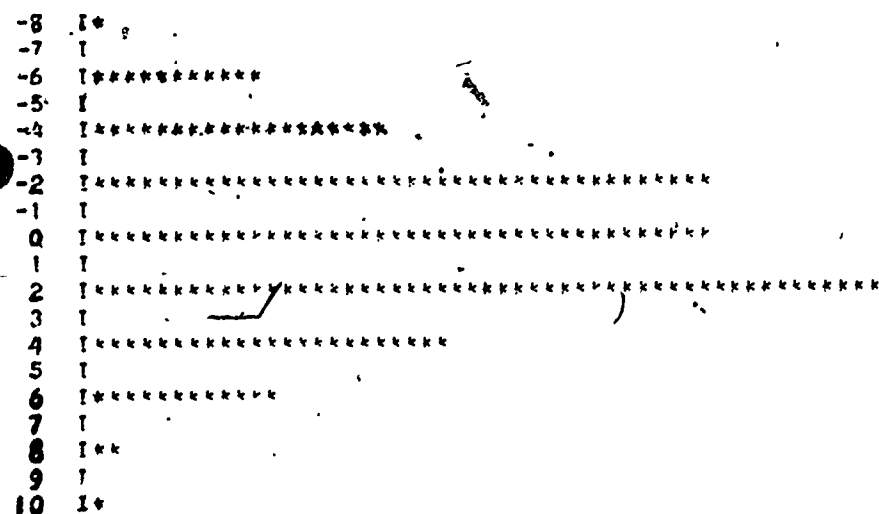


Figure 5. Random walk simulation for the case of equal probabilities of moving left or right, obtained using the program RAND.

He is asked as part of the same exercise to state the most frequent end point for each walk length, how this value changes as the walk length increases, and what happens also to the shape of the histogram as the walk length increases. After recording these observations, he is required to repeat the same sequence of walks three more times for the probabilities 0.3, 0.6 and 0.7 of moving to the right, and to compare the results (a sample of which is shown in Fig. 6) with those obtained for the first exercise. From his results, he should learn that for the first case of equal probability (for a move in either direction), the most frequent end point is in the vicinity of the starting point for any step length, but the distribution spreads as the step length is increased. With unequal probabilities, there is a shift of the most probable end point in the direction of higher probability, and this shift increases as the number of steps increases.

LENGTH OF EACH WALK? (BETWEEN 2 AND 100). ?10
PROBABILITY OF A MOVE TO THE RIGHT? (BETWEEN 0 AND 1)?0.3
NUMBER OF RANDOM WALKS TO RUN? (FROM 1 TO 9999)?200

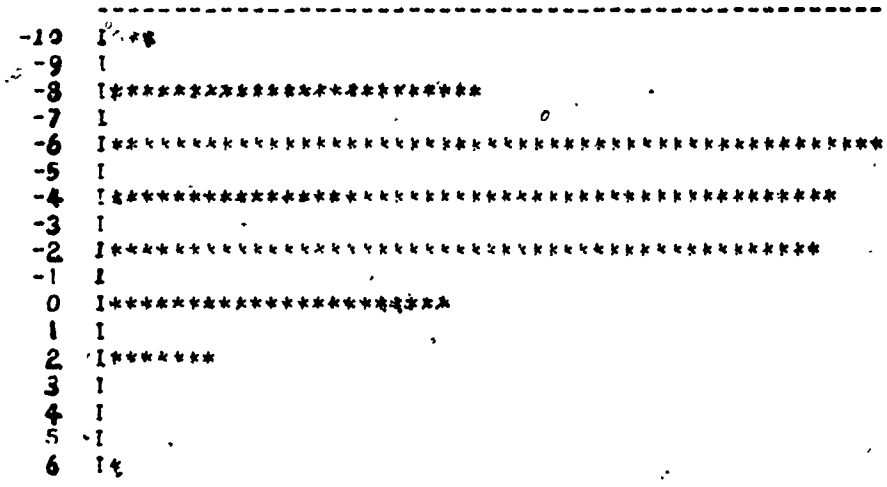


Figure 6. Random walk simulation for the case of greater probability in moving to the left, obtained using the program RAND.

The consequences of this shift for unequal probabilities become far more apparent when the student is asked to consider a one-dimension field of 31 positions labeled from 0 to 30, and for a mover to be placed at the midpoint (position 15) with a probability of $2/3$ for moving a unit distance to the right, and of $1/3$, to the left. He then is asked what are the probabilities of reaching the two end points (30 and 0). The student may do this quite easily in an exercise which uses the program RWALK, a Monte Carlo simulation of the situation described, but in the form of a series of games. A game terminates when either end point is reached, with a "win" attributed to that particular result; a win for "the other team" occurs when the opposite end point is reached in some other game. The student must input the number of field positions (30), the starting position (15), and the probability of moving toward zero (0.333333). Figure 7 shows the outcome of having the computer execute 100 games, with the "playing" of one game illustrated. Any number of the games may be displayed in this fashion, but it is unnecessarily time consuming to call for more than a couple of them. I have deliberately referred to "the outcome" above, because even though each game is statistically independent of every other game, there are statistically overwhelming odds that the outcome of 100 games (with the given input) would result in a score of 100 to 0. An additional exercise using RWALK requests that the user find what starting position will result in an equal probability of reaching either end (with the same probabilities of $2/3$ and $1/3$ for steps to the right and left). The usually surprising result is that one must start in field position 1 to insure something even close to a score of 50 to 50 for 100 games. Any higher starting position provides significant bias to the contest. The significance of these startling results are then related to the random motion of electrons in a current-carrying wire.

Unit 2, "The Approach to Equilibrium and Equilibrium Fluctuations," is essentially aimed at showing how the microscopic statistical behavior of a system, determines its macroscopic state, although the stated OBJECTIVES are presented in much greater detail. PREREQUISITES consist only of a familiarity with the molecular nature of gases, i.e., that they are composed of "tiny" molecules moving at high speeds and have relatively large average distances between them. The INTRODUCTION briefly recalls that we expect to find about 10^{19} molecules per cubic centimeter of air. More importantly, it states that the results obtained from statistical computation do not tell us "how the system must behave, but instead, how it is most likely to behave." The DISCUSSION begins by enumerating the number of different ways to arrange four labeled molecules into two initially empty containers. The table that follows is identical to that in the module text.

A	B	A-B	# Ways
1 2 3 4		4-0	(1)
1 2 3	4		
1 2 4	3	3-1	(4)
1 3 4	2		
2 3 4	1		
1 2	3 4		
1 3	2 4		
1 4	2 3	2-2	(6)
3 4	1 2		
3 2	1 4		
2 4	1 3		
1	2 3 4		
2	1 3 4	1-3	(4)
3	1 2 4		
4	1 2 3		
	1 2 3 4	0-4	(1)

As an exercise, the student is requested to perform the same "cataloguing" operation for an increase to 6 molecules. The ramifications of this change are discussed, pointing out that the probability of finding an equal or near equal distribution of molecules in the two boxes increases quite dramatically as the number of molecules becomes considerably greater. At this point, the user is presented with the basis for another computer simulation, EHREN. In it, we start with two containers of equal volume, initially with one empty and the other with an arbitrary number of gas molecules. The computer "chooses" a molecule at random and "moves" it from the container it currently occupies to the other one. This operation is repeated a specified number of times. If that number is large enough, we expect to find an equilibrium distribution of about equal numbers in each container. The results of this simulation, which the user must call using several sets of input, can be obtained in graphical or tabular form. Figure 8 shows a sample of the former.

RWALK
 WHAT DO YOU WANT TO NAME THE FIRST TEAM? EINSTEIN
 THE SECOND TEAM? NEWTON
 HOW MANY STEPS DO YOU WANT? (MUST BE >2) 31
 WHAT DO YOU WANT FOR THE STARTING POSITION? 15
 HOW MANY OF THE FIRST 100 GRAPHS DO YOU WANT PRINTED? 1
 AND THE PROBABILITY OF MOVING TOWARD ZERO? .333333

---STARTING POINT-----> 15
 ---PROBABILITY-----> .333333
 ---GAME MOVES-----> 28

EINSTEIN NEWTON
 0 5 10 15 20 25 30
 H + + + + + H

-----> NEWTON WON.

-----> EINSTEIN WON 0 GAMES OUT OF 100.
 -----> NEWTON WON 100 GAMES OUT OF 100.
 -----> 0 GAMES TOOK OVER 1000 MOVES.
 DO YOU CARE FOR ANOTHER GAME? NO

Figure 7. Sample run of the program RWALK.

ENTER THE NUMBER OF PARTICLES IN 'A' AND
 THE NUMBER IN 'B', SEPARATED BY A COMMA? 1000,0
 ENTER THE NUMBER OF CYCLES TOTAL, AND THE
 NUMBER OF CYCLES PER REPORT, SEPARATED BY
 A COMMA? 3000,100
 DO YOU WANT NUMERIC OR GRAPHIC OUTPUT? GRAPHIC

THE EHRENFEST GAME:
 TOTAL NUMBER OF PARTICLES = 1000
 CYCLES PER REPORT = 100

	I	'A'	I	'B'
100	I	IAAAAAAAAAAAAAAAAAAAAAA	I	198
200	I	IAAAAAAAAAAAAAAAAAAAAAA	I	18898
300	I	IAAAAAAAAAAAAAAAAAAAAAA	I	199388
400	I	IAAAAAAAAAAAAAAAAAAAAAA	I	18899888
500	I	IAAAAAAAAAAAAAAAAAAAAAA	I	18899988
600	I	IAAAAAAAAAAAAAAAAAAAAAA	I	198999888
700	I	IAAAAAAAAAAAAAAAAAAAAAA	I	1883899888
800	I	IAAAAAAAAAAAAAAAAAAAAAA	I	1888398889
900	I	IAAAAAAAAAAAAAAAAAAAAAA	I	19889938889
1000	I	IAAAAAAAAAAAAAAAAAAAAAA	I	19899888399
1100	I	IAAAAAAAAAAAAAAAAAAAAAA	I	19993888988
1200	I	IAAAAAAAAAAAAAAAAAAAAAA	I	199988838888
1300	I	IAAAAAAAAAAAAAAAAAAAAAA	I	198899838888
1400	I	IAAAAAAAAAAAAAAAAAAAAAA	I	198888838888
1500	I	IAAAAAAAAAAAAAAAAAAAAAA	I	188899838888
1600	I	IAAAAAAAAAAAAAAAAAAAAAA	I	188999888888
1700	I	IAAAAAAAAAAAAAAAAAAAAAA	I	188999888888
1800	I	IAAAAAAAAAAAAAAAAAAAAAA	I	188848888888
1900	I	IAAAAAAAAAAAAAAAAAAAAAA	I	198899888888
2000	I	IAAAAAAAAAAAAAAAAAAAAAA	I	188999888888
2100	I	IAAAAAAAAAAAAAAAAAAAAAA	I	188888999888
2200	I	IAAAAAAAAAAAAAAAAAAAAAA	I	188888889888
2300	I	IAAAAAAAAAAAAAAAAAAAAAA	I	188899888888
2400	I	IAAAAAAAAAAAAAAAAAAAAAA	I	188888388888
2500	I	IAAAAAAAAAAAAAAAAAAAAAA	I	188888888888
2600	I	IAAAAAAAAAAAAAAAAAAAAAA	I	188888888888
2700	I	IAAAAAAAAAAAAAAAAAAAAAA	I	183888888888
2800	I	IAAAAAAAAAAAAAAAAAAAAAA	I	188888888888
2900	I	IAAAAAAAAAAAAAAAAAAAAAA	I	188888888888
3000	I	IAAAAAAAAAAAAAAAAAAAAAA	I	183888888888

WOULD YOU LIKE TO CONTINUE FOR MORE CYCLES? NO
 WOULD YOU LIKE TO TRY OTHER INITIAL CONDITIONS? YES

Figure 8. EHRENFEST output for 1,000 particles placed initially in container A, with none initially in container B.

I shall suspend further detailed exposition of the module at this point in the interest of brevity. However, I do wish to mention that the relationship between sample size and fluctuations is fully exploited. Unit 3, "The Boltzmann Factor and The Law of Atmospheres," uses a program ATMOS which is similar in many respects to EHREN. Figure 9 provides a sample of partial output from ATMOS. I'd also like to point out the presence of some optional discussion sections which treat the subject material with mathematical rigor appropriate to upper class undergraduates or at least for students enrolled in a calculus-level introductory physics course.

I shall not try to analyze the shortcomings of our physics modules because I am too closely associated with them to see these as clearly as others. Our major problems are, as already indicated, a possible non-uniformity of level of presentation, and subject matter which may not have universal applicability.

6.3 Mathematics

The mathematics participants initially set out to produce modules (for the most part) applicable to the elementary calculus curriculum and for use by students of mathematics, the physical sciences and engineering. Certain of these modules, as mentioned earlier, cover material not even worth mentioning if it were not for computer access, e.g., numerical methods for integration and the solution of differential equations. Most of the modules falling into this category deal with approximation methods that rely heavily on the computer solely as a computational tool. Although real physical, biological, social and economic applications are stressed in all modules, the computer use of the actual mathematics invoked is, of necessity computational. Corresponding the use of simulation as an educational aid has been sparse when compared to such use for chemistry and physics. I don't necessarily agree, however, that discovery-type simulations of physical events should be avoided altogether, particularly since rigorous mathematical proofs may be included as well. Yet, it seems particularly difficult to convince our mathematicians to take anything but a conservative approach to computer-augmented education. I believe too, that they may well be representative of mathematicians everywhere, so that they may be quite practical in disregarding my suggestions for more creative uses. A similar situation seems to apply to chemists, who seem to use the computer quite often simply for drill-and-practice operations, particularly since so much of introductory chemistry is empirical in nature.

Midway through the current project, the mathematicians recognized a much greater need and market for computer-oriented materials in what might be called "business mathematics," even

```

INPUT NO. OF PARTICLES(N1) UP TO 500 AND
NO. OF COLLISIONS (N2)
7100,10000
INPUT INITIAL ENERGY/PARTICLE (SUGGESTED RANGE 1-5)
?5
INPUT NUMBER OF PLOTS DESIRED
?10
DO YOU ALSO DESIRE A TABULAR DISPLAY?NO
100 PARTICLES
DENSITY DISTRIBUTION AFTER 0 COLLISIONS
G'S REPRESENT PARTICLES IN THE GROUND LEVEL

```

```

-----
0 I
1 I
2 I
3 I
4 I
5 I<<<<< ALL PARTICLES ON THIS LEVEL >>>>>
6 I
7 I
8 I
9 I
10 I
11 I
12 I
13 I
14 I

```

```

100 PARTICLES
DENSITY DISTRIBUTION AFTER 10000 COLLISIONS
G'S REPRESENT PARTICLES IN THE GROUND LEVEL

```

```

-----
0 IGGGGGGGGGGGGGGGGGG
1 IXXXXXXXXXXXXXXXXXX
2 IXXXXXXXXXXXXXXXXXX
3 IXXXXXXXXXXXXXXXXXX
4 IXXXXXXXXXXXX
5 IXXXXX
6 IXXXX
7 IXXXXXX
8 IXXXXXX
9 IXXXX
10 IXX
11 IXX
12 I
13 IX
14 IXXX

```

Figure 9. ATMOS output for 100 particles starting at level 5.

though some of the methods covered have equal validity (but lesser importance) for science students. To aid them I brought them in contact with the Dean of IIT's Stuart School of Business and Finance, himself a devotee of the computer in solving industrial problems. He stressed the need for an update on the mathematical methods being taught to his students and pointed out that typically about 500,000 students are enrolled in "business math" courses each year in the U.S. This information, and the earlier convictions of some of the mathematicians themselves, did lead to a strong effort to develop business-oriented modules. A good example of this is the one entitled "Solving Linear Systems of Equations Using Gaussian Elimination," a first version of which had appeared in the initial stage of the project.

The Gaussian Elimination module can, in fact, be used by a student to learn this method without recourse to any computer usage. Nevertheless, the method (involving matrix manipulation) is taught via an algorithmic approach; indeed, it is referred to as the Gaussian elimination algorithm. The student is then told that he could save a great deal of effort in solving large systems of linear equations by applying this algorithmic approach to the writing of a computer program. He is led through the programming on a step-by-step basis, in a sense reinforcing the understanding of the algorithm used earlier [This module, unlike most others does presume that the student has had a minimum of a 2 to 3 hour discussion on programming in BASIC or FORTRAN IV - hardly an overwhelming requirement]. The total program is separated into four subprograms called NONZERO FINDING, SWITCHING, NORMALIZING, and BLASTING. Each subprogram performs a single mathematical task on a matrix as follows:

NONZERO FINDING locates the first row, starting at a specified row in a given column, for which the entry is nonzero;

SWITCHING interchanges two rows;

NORMALIZING reduces the coefficient of a diagonal element to 1;

BLASTING reduces non-diagonal elements to 0.

Well-annotated flow charts are included for each subprogram, as well as the initial programming statements for given test matrices.

The module entitled "Markov Chains" is one in which the computer is used only for its calculational ability. Both regular and absorbing Markov chains are introduced by presenting some interesting "real-life" problems. In the former instance, the problem involves determining the percentage of the ultimate

market for the product of each of four manufacturers, given certain buying patterns. The latter instance involves a determination of the long term population of assistant, associate and full professors at a university, given certain patterns of mobility and transition. These problems are not only of some interest, but provide a model and "raison d'être" for the study of Markov chains. The computer is used only to handle assigned exercises in which it is necessary either to find some high-order power of a matrix or the inverse of the identity matrix minus a given matrix. I personally feel this module is well-structured and presented, but I feel also, that it could possibly be enhanced by the addition of a simulation which helps establish a feeling for the probability of various events upon which the analysis involving matrix operations is based. I have been informed, however, the approach chosen was deliberate (and is symptomatic of other modules in this discipline) to make the module more palatable for potential publishers.

The project mathematicians believe that their business modules will provide in themselves an entire curriculum, worthy of adoption "in toto" and possibly in book form. Only time will tell if this judgment is correct. In the interim I have exhorted them to gain as much experience as possible with their own students. They feel, on the other hand, that their calculus-related modules are more likely to see piece-wise adoption because these do not constitute the basis of an entire curriculum.

7. EVALUATION

The past academic year is the first in which significant actual testing of the modules has been carried out in the only really meaningful way - through student use. Previously, embryonic versions of the modules did receive some student-testing, but generally this was done only by a given module's originator. Unfortunately, this is still the fate of some modules which have just come to fruition during the past year. The major use and evaluation took place for the chemistry modules at three institutions: Virginia Military Institute, Western Carolina University and Xavier University. Physics modules were used at Pima (community) College and my own institution, IIT, while I believe the only (non-author) use of some mathematics modules occurred at Carnegie-Mellon University. These are all institutions represented by participants in our project, although some module adoptions were made by the participants' colleagues. Generally, there were no more than twenty to thirty students involved at each university.

At the same time the modules were first distributed, a student evaluation form and an instructor's evaluation were produced. As most instructors using the modules so far have been our own participants, we were concerned mainly with student response. Unfortunately, some of our participants failed through oversight to have their students fill out the forms, although less formal evaluations have been reported. However, we did obtain valuable feedback from the three universities using all or most of the chemistry modules, and from IIT, where a colleague and I used a total of three physics modules. In every case, a fundamental error was made; namely, the students were asked to fill out and return the evaluation forms at their convenience. As a result, completed forms averaged about 50% of student enrollment at each reporting institution, combining to make a total of less than 100 responses. Despite this sad record, there were many common responses, independent of discipline; and in the case for chemistry where the entire collection of modules were used in several classes, a few less-well received modules were identified.

The student evaluation form asked the student to rate his degree of agreement with three positive statements:

1. This module (or unit) helped me understand the subject matter better.
2. The module was well worth the effort I put into it.
3. I would prefer to use a complete set of modules instead of regularly scheduled lectures.

The student evaluators were almost unanimous in agreeing with the first two statements, and were for the most part in disagreement with the last. We are naturally encouraged by the first two responses, since a negative response would be a clear signal of failure (although it is known that positive statements usually elicit positive responses). We are not really too concerned about the last response, because in no case was a completely self-paced mode of instruction employed, and I strongly suspect, as well, that almost all the students involved had never had any experience with this type of presentation. Furthermore, self-paced instruction does not preclude occasional lectures being offered, and additionally, students were not offered any intermediate alternatives, e.g., a course involving both lectures and modules.

The fourth question asked students to rank in order of effectiveness (for the material covered in a given module), light different educational media, including one referred to as "other students." Not too surprisingly, lecture or lecture

demonstration obtained the highest consensus ranking, but most encouraging was the fact that the modules outranked a conventional textbook.

Additional questions asked for a report on the total time and terminal time used for the module in question. Responses were quite uniform (although not completely so) and within the limits anticipated by the module authors. A request for specific comments yielded some which will be of value when the next editions of the modules are prepared.

I can summarize our entire evaluation input by saying that it has served to reinforce the strength of our convictions in proceeding to produce additional modules and to revising many of the existing ones. We realize, however, that we must improve the quality of our evaluation procedures and collect a far greater sample of student and faculty opinion.

8. A LOOK INTO THE FUTURE

It is far too early to forecast the ultimate success of our; as yet, unfinished efforts. At the moment, additional external funding for additional activities and for dissemination of our modules is uncertain. But whatever the outcome, we intend to pursue our goal of producing materials which are easily usable and desired by a large segment of the academic community, and as a result, to interest a reputable publisher to purchase the rights to our finished products.

We realize that more immediately we must have greater feedback from both students and faculty. To this end I have charged the participants in this project with seeing that our modules are used more fully on their campuses. We also have sent out, either free or at cost, sets of the modules to about 100 to 200 individuals in each discipline during the past year. We are only now catching up with requests still coming in as a result of a general mailing and articles or notices in professional journals. I am encouraged that at least one institution not represented by project participants is already planning to use our chemistry modules with a class of 20 to 25 students during the 1976-1977 academic year, and that Xavier University will "go all the way" at the same time in using the chemistry modules with an estimated 175 students. Although one member of the Xavier Chemistry Department is one of our participants, the decision to use the modules was made by his four colleagues. Other commitments to adopt our modules are believed to be forthcoming, and any inquiry as to module availability is welcome at this time.

If our specific efforts do not result directly in the desired publication of our ultimate efforts, I will not despair. Through our public relations efforts, we are exposing the academic community to a particular type of framework and approach to utilizing the medium of computer-based education. I believe that some elements of what we are doing, along with elements of other presentations in this volume, shall survive to enhance and reduce the cost of the educational process. With this thought in mind, I can only be proud of our efforts thus far, and continue to press onward with an open mind and renewed determination.

9. ACKNOWLEDGEMENTS

As mentioned so often, the project is a collaborative one, and credit should go to that entire dedicated group of faculty participants whose names appear in the Appendix beside the units and modules that they have authored. Special thanks go to the chemistry and mathematics group leaders, Professors Cynthia Jameson and Joseph Mayne for efforts well beyond the call of duty. My various secretaries and student programmers have also contributed significantly to making this project a reality. Finally, but certainly not least, I wish to pay tribute to a man who not only was among the first to recognize the educational potential of the computer, but has been helping me continually with ideas and encouragement since my entry into the field almost a decade ago, Professor Alfred Bork.

REFERENCES

1. The author was involved with one of the first of these, which featured remote teletype access from twelve colleges to the IIT Computer Center.
2. A computation of the costs of university classroom instruction made about 5 years ago indicated about \$5 to \$6/hour. Computer-based education is comparable to this.
3. Edited by R. Blum (Comm. on Col. Physics 1968), available from Information Pool, American Institute of Physics, SUNY, Stony Brook, N. Y. 11790.
4. This style was introduced by Lillian Lieber in The Education of T. C. Mits (The Common Man in the Street).

APPENDIX

I. PHYSICS MODULES

- A. Impulse and Momentum
Dan Davidson
Pima College

Three programs are utilized in helping students to understand these fundamental physical quantities, their relation to Newton's laws of motion, and the conservation of momentum. The programs involve the increase in velocity of a body as "impulse units" are added, conservation of momentum calculations, and the behavior of a rocket ship in a gravity-free environment.

- B. Circular and Orbital Motion
Lawrence E. Turner, Jr.
Pacific Union College

This work provides an introduction and discussion of circular motion, beginning with the Kinematics of uniform circular motion, and including several applications, one of which is circular satellite orbits. The orbital motion is then generalized to include the elliptical situation.

- C. Wave Motion
Harold Weinstock, IIT, and
Samuel L. Wiley, Cal. St. Col., Dominguez Hills

A complete introduction to wave motion is presented. Students access a series of programs which permit the addition of sine waves, with output available in graphical or tabular form. Problems involve the principle of superposition, the phenomenon of beats, Fourier synthesis, and standing waves.

- D. Interference and Diffraction
Dan Davidson
Pima College

Two programs are used to simulate the effects of interference and diffraction. Concepts are developed, gradually increasing in sophistication from double slit interference to multiple slit gratings with diffraction. Fresnel diffraction is dealt with first, and Fraunhofer diffraction is treated as a special case.

E. Geometric Optics

Dan Davidson, Pima College
and Harold Weinstock, IIT

A complete introduction to refraction (and some discussion of reflection) is presented. Beginning with a computer game illustrating Fermat's principle, the student uses programs involving Snell's law and total internal reflection, refraction of light through a prism, and two lens optical systems.

F. Electrostatics

Philip Goldstein
Jersey City State College

Unit 1 - Coulomb's Law. This unit includes a discussion of basic electrostatic phenomena and their interpretation in the light of atomic theory. Brief glimpses into the historical development of electric theory are included. The computer is used to compute forces between charges.

Unit 2 - The Electric Field. Topics covered are the concept of electric fields, field maps and field lines, two important charge distributions and the motion of particles in electric fields.

Unit 3 - Electric Potential. After a discussion of electric potential and potential energy, and the relation between electric field and potential, the computer is used to plot the two-dimensional equipotential surfaces of specific charge distributions.

G. Magnetism and Charged Particle Motion

Harold Weinstock
Illinois Institute of Technology

Students utilize a program which allows them to specify uniform magnetic and electric fields, and the initial velocities of particles. Output is in the form of drawings of two dimensional projections of the motion and of the energy vs. time relationship.

H. Transient and Alternating Current

Dan Davidson
Pima College

Several computer programs are used to demonstrate graphically DC transient and AC circuit response. Effects of resistors, inductors and capacitors are discussed. These are then related to phase, reactance and inductance in AC circuits.

I. The Principle of Minimum Energy

Wayne Lang
MacMurray College

The gravitational potential energy between two masses is introduced and then expanded to the potential energy sum for many masses. Two expressions for this sum are developed and a computer program based upon one of these expressions is exhibited. Other applications of the minimum energy principle involve crystalline structure, the shape of a water drop and nuclear energy.

J. Statistical Properties and the Behavior of Gases

Samuel L. Wiley, Cal. St. Coll., Dominguez Hills,
and Harold Weinstock, IIT

An introduction is given to properties associated with random processes in many-particle systems. Students use computer simulations to observe the effect of randomness and the statistical behavior of collections of particles of varying size. Topics covered include random walk, gaseous diffusion and energy distributions.

K. Temperature and Thermal Equilibrium

Samuel L. Wiley
California State College, Dominguez Hills

Computer simulations are used to demonstrate the behavior of systems in thermal equilibrium. The heat content and temperature of a gas are related to the kinetic energy of its component molecules and it is shown that thermal equilibrium is established between two systems when their average molecular kinetic energies are equal.

L. Radioactive Decay and Nuclear Processes

Lawrence E. Turner, Jr.
Pacific Union College

This module presents the concept of radioactive decay beginning with the statistical aspects of the decay events. Nuclear fission is discussed with simulations that consider critical mass configurations, moderation, and reactors.

II. CHEMISTRY MODULES

A. Modular Appendices

1. Exponential Numbers and Logarithms - C. Jameson
2. Use of Conversion Factors - R. Williams
3. pH, Strong Acids and Bases - F. Settle
4. Naming of Chemical Compounds - A. K. Jameson. A hangman game (NAME) on the computer introduces the student to names of elements. The system of chemical nomenclature is discussed in the manual. The program NOMEN presents the student with chemical formulae. He types in the names and is judged interactively. Nomenclature is normally not included in a lecture, but is assigned to students. This Appendix suffices as a sole source.

B. Atomic Structure

A. K. Jameson
Loyola University

The classic experiments leading up to the Rutherford model of the atom are discussed. The program MLKAN simulates the Millikan oil drop experiment.

C. Stoichiometry

Unit 1: The Mole Concept.
Robert C. Williams
University of Nebraska

This unit deals with problems involving the mole concept, elemental composition and determination of empirical formulas. The programs used are 3 drill programs: MOLE, PERCOM and EMPIR, which provide problems and check student solutions to them.

Unit 2: Solving Stoichiometry Problems.
Robert C. Williams
University of Nebraska

This provides problem-solving practice with tutorials and drill on simple stoichiometry, limiting reagent and percent yield problems.

D. Gases, Liquids and Solids

Unit 1: P-V-T Behavior of Gases
C. Jameson
University of Illinois - CC

A program, GASES, simulates the behavior of some gas trapped in a cylinder with a piston. The student is given the initial

conditions. He varies the temperature. A picture of the system is shown at each temperature. The data (volume and temperature) are then tabulated. The student is also allowed to vary the pressure at constant temperature, and the data (volume and pressure) are tabulated for him. In the manual, the compulsory exercises ask the student to plot P and V, V and 1/P, V and t ($^{\circ}\text{C}$). He is to find the zero volume intercept and the conversion from Celsius to Kelvins. Ideal gas law problems are also discussed.

Unit 2: Phase Equilibria
M. Bader
Moravian College

A standard discourse with problems interspersed, but without any computer usage, is involved.

E. Solutions

Unit 1: Introduction to Solutions
M. Bader
Moravian College

This uses a "2-pass" quiz program. The first pass generates a quiz which a student takes off-line. In the second pass, the student enters his answers and the quiz code number. This code number, generated randomly on the first pass, acts as a key to obtaining the correct answers to and subsequent grading of the quiz.

Unit 2: Molarity, Molality and Concentration Conversions
M. Bader
Moravian College

Molarity and molality are discussed and examples are worked out in detail. The student then has a chance to take a "2-pass" quiz to test his knowledge of the subject matter.

Unit 3: Dilution Problems
Richard T. O'Neill
Xavier University

This uses a tutorial program, DILUTE, which diagnoses errors in setting up dilution problems.

Unit 4: Solution Stoichiometry
Richard T. O'Neill
Xavier University

Program ENDPT simulates a titration (in a game format) as an aid to solving problems of reactions in solution. Acid-base and oxidation-reduction reactions are used.

Unit 5: Colligative Properties of Solutions
M. Bader
Moravian College

This unit deals with colligative properties in a standard fashion, but uses a computer to generate two sets of quizzes of varying levels of difficulty.

F. Chemical Equilibrium

Unit 1: Introduction to Chemical Equilibrium
John J. Manock
Western Carolina University

This uses two programs which allow the student to discover the nature of chemical equilibrium and the equilibrium constant. Program EQSIM involves the discovery of chemical equilibrium by "teletype" graphics which show that equilibrium is dynamic, not static. It further portrays the relationship between the numbers of different species of molecules. Program KEQ aids in the discovery of the functional form of the equilibrium constant. The program, providing simulated data for various initial conditions, allows a student to input his choice of concentration term combinations until he finds one which is function invariant with respect to initial conditions.

Unit 2: LeChatelier's Principle
A. K. Jameson
Loyola University

This uses simulation of a chemical system in equilibrium by "teletype" graphics. A student may disturb equilibrium by addition of reactants or products, or changing pressure. The program simulates how the system reacts, showing before and after pictures.

Unit 3: Chemical Equilibrium Calculations
C. Jameson
University of Illinois - CC

Program EQUIL allows a student to find equilibrium concentrations a trial and error approach. This is done by comparing the

values of concentration terms with the equilibrium constant at each stage. The student must choose the direction the reaction is required to take in order to reach an equilibrium state. The Student Manual includes algebraic methods for the solution of such problems. Program EQCALC uses the same approach on a student's choice of reaction and initial conditions for systems too difficult to handle algebraically because of the high-order polynomial equations involved. Program EQUIZ is a "2-pass" quiz-generating program which uses a random selection of systems and data; and on the second pass, given the quiz code number, it will print only the answers to any quiz generated.

Unit 4: Equilibrium in Acid-Base Systems
Frank A. Settle, Jr.
Virginia Military Institute

This unit uses three programs to explore acid-base equilibria. Here a student (1) discovers how pH and the degree of dissociation changes with the analytical concentration of an acid; (2) finds the limits of applicability for a commonly used approximation-
tedius, but precise, calculations are done by execution of the program, while the student makes cruder hand calculations and then compares the two results; (3) discovers how a buffer system reacts to certain changes. The student inputs these changes for which new pH values are calculated.

Unit 5: Heterogeneous Ionic Equilibria
John J. Manock
Western Carolina University

A computer program is used to simulate laboratory data that would be obtained for a reaction involving heterogeneous equilibria.

G. Redox and Electrochemistry

Unit 1: Oxidation Number and Balancing Redox Equations
Richard T. O'Neill
Xavier University

Balancing of oxidation-reduction equations by both the oxidation number method and the half-reaction method is discussed in the manual. The program OREBAL helps a student balance ionic redox equations in a step by step fashion. A very large number of possible examples may be picked since half-reactions rather than whole equations are stored.

Unit 2: Electrolysis
M. Bader
Moravian College

This unit enables the student to predict the products of electrolysis of molten and aqueous electrolytes and to predict quantitatively the amount of reaction in an electrolysis cell using Faraday's laws. The program FARADAY is about a problem involving the application of Faraday's law in a spy-thriller context.

H. Chemical Kinetics

Unit 1: Reaction Rates and Routes
Frank A. Settle, Jr., V.M.I., and
John J. Manock, Western Carolina U.

This unit presents the half-life method, the graphical method and the method of initial rates, with emphasis on the latter in determining the rate equation for a reaction. The program KINET selects a rate law for the reaction $A + B \rightarrow C + D$. The program also generates a unique set of initial concentrations for A and B and the initial rate observed. The student conducts experiments by varying the initial concentrations of each reactant to be able to find the order of the reaction with respect to A and B. The program then proposes several mechanisms for the reaction. The student chooses one which is applicable. If he selects an incorrect mechanism, he is shown the rate equation which would have been observed if the mechanism had been the one he picked.

Unit 2: Effect of Temperature and Catalyst on Reaction Rate
Frank A. Settle, Jr., V.M.I., and
John J. Manock, Western Carolina U.

This unit introduces the concept of activation energy and the mode of action of a catalyst. The program PILOT illustrates the role of temperature and catalyst in the design of a pilot plant for an industrial process. A company wants to produce a new polymer at a given optimum rate for reasons of safety (not too fast) and economy (not too slow). In the design of the plant the student must vary the temperature and select a catalyst while considering the cost factors. Each student will have his own unique set of parameters. From these the teacher can calculate the optimum solution (minimal cost) and compare the student's results with it. The objective is to make the student aware of the non-linear dependence of the rate on the temperature and the effect of a catalyst on the rate.

III. MATHEMATICS MODULES

A. Elementary Numerical Solutions for Ordinary Differential Equations

Unit 1: Richard A. Alo
Lamar University

Simple Euler and Euler predictor-corrector methods are introduced and discussed. Elementary analytical and numerical solutions of first order differential equations are obtained and the results are applied to some examples from chemistry and physics.

Unit 2: L. Carl Leinbach
Gettysburg College

In this unit the fourth order Runge-Kutta method is applied to the example from Unit 1, and the results are compared with those of the earlier methods and of the analytical solutions. Finally, the "leap-frog" method is used to solve a second order equation with given initial values.

Unit 3: L. Carl Leinbach
Gettysburg College

The methods of Unit 1 and Unit 2 are extended for use with systems of equations. Special emphasis is placed on the utility of computer models with respect to ease of modification. The unit concludes with a discussion of how higher order equations may be viewed as a system of first order equations.

B. Approximation of Functional Values by Polynomials
Richard A. Alo, Lamar University, and
L. Carl Leinbach, Gettysburg College

This module deals with the Taylor polynomial approximation of functional values and applications of such approximations to integrations, differential equations and functions such as the arcsin and arctan.

C. Mathematical Modeling (Calculus Version)
L. Carl Leinbach
Gettysburg College

Newton's Laws of Motion are presented as a paradigm of a mathematical model. Then Verhulst's population growth model, a model for the predator-prey problem, as well as models for the spread of epidemics and for predicting industrial pollution are developed with an emphasis on how such models may be computer implemented.

- D. Numerical Integration
Joseph H. Mayne
Loyola University

Starting with an intuitive discussion of area, this module introduces methods for approximating the definite integral of a continuous function. The methods are then applied to a gas chromatography experiment and a problem of electrical power generation.

- E. Linear Programming
Lyle E. Mauland
University of North Dakota

Unit 1: After seeing some examples which require the maximization of a linear expression subject to linear constraints, the student is introduced to the simplex method for solving this type of problem. Then the student is guided in the development of a computer program which uses the simplex method.

Unit 2: Variations on the problem considered in Unit 1 are considered, as well as the dual problem of minimization. Also included is a discussion of the limitations of the simplex method and an indication of directions for further exploration.

- F. Gaussian Elimination
J. C. Mathews
Iowa State University

After carefully considering the problem of solving small systems of linear algebraic equations, this module introduces the Gaussian elimination algorithm. Flow charts for writing a program to implement the algorithm are developed and a complete program is provided in the appendix.

- G. Error Analysis
Noal C. Harbertson
California State University, Fresno

This module introduces the definition of absolute and relative error and discusses the sources of error in mathematical modeling and numerical computation. Elementary error propagation and simple examples of roundoff and truncation are included.

- H. Fourier Analysis of Linear Programs
Richard A. Alo
Lamar University

This involves the solution of linear programs using pairwise elimination. For small programs the method is a great

simplification over the simplex algorithm. The Fourier method also permits parametric analysis which is not possible with the standard simplex method.

- I. Basic Trigonometry
Richard A. Alo
Lamar University

This module gives a basic introduction to angular measure, circular functions, and fundamental relationships. Some applications to the physical and economic sciences are considered.

- J. Transportation Problem
Noal C. Harbertson
California State University, Fresno

The transportation problem is presented by means of an example, and the stepping algorithm is discussed. Included are techniques for setting up the transportation tableau and finding an initial feasible solution.

- K. Mathematical Modeling (Non-calculus Version)
L. Carl Leinbach
Gettysburg College

Starting with an intuitive discussion of rates of change and applying the ideas to the falling body problem, this module develops models for financial analyses, Samuelson's model of national income, an elementary queuing model, and a diet model. All the models are computer implemented and the reader is encouraged to experiment with the parameters of each model.

- L. Geometric Programming
Richard A. Alo
Lamar University

Unit 1: Methods for handling geometric programs are discussed with applications to business problems.

Unit 2: Elementary business examples are analyzed with the arithmetic-geometric inequality. Optimization problems with constraints are presented.

- M. Matrix Inversion, Determinants, and Cramer's Rule
J. C. Mathews
Iowa State University

The Gaussian elimination algorithm is modified to give an efficient computer program for matrix inversion. Determinants are explained together with Cramer's rule for the solution of

linear equations. There are extensive problem sets throughout.

M. Probability

J. C. Matheva

Iowa State University

Unit 1: The first unit discusses counting techniques, the idea of a sample space and assignment of probabilities to elementary events, and the calculation of the probability of the union of two events. In addition, Bernoulli trials are introduced.

Unit 2: The topics of independent events, conditional probability, and the formula of Bayes are taken up. This is followed by an elementary discussion of expected value.

O. Vectors and Matrices

Joseph H. Mayne

Loyola University

Elementary algebra of vectors and matrices is introduced. The matrix multiplication algorithm for square matrices is developed and the inverse of a matrix is discussed.